Language, Notation & Meaning:
Children's articulation of their emergent fraction ideas

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Abstract:

This research focused on children's use of academic language when discussing fractions while solving equal sharing problems. It also focused on their mathematical thinking related to fractions. An understanding of fractions is important to lay a foundation for students to understand more advanced mathematical concepts. Research indicated that if fractions are to be learnt from a deeper understanding, equal sharing problems can serve as a strong foundation. In order to conduct this research, we analyzed the mathematical language used by 3 to 10 second-fifth graders (ages 7 to 12) in interview contexts. The aim was to see how children use language to express their interpretations of fractions. The hypothesis was that children tend to be imprecise when talking about fractions which can lead to misconceptions (Grapin et al., 2019). This analysis was intended to provide teachers with examples to consider so they can do a better job of supporting students in learning the language and concepts of fractions.

Introduction:

The focal point of this study was the usage of children's language while discussing fractions. Fractions are the building blocks for students, which will help them recognize the properties of ordering real numbers on a number line (Seigler & Fazio, 2013). Not only children, but adults as well have difficulties with fractions. A study had shown that students that were fractions poorly predicted a mediocre quality of fraction knowledge as they moved forward in their academia (Seigler & Fazio, 2013). Studies have shown that equal sharing can provide a foundation that is robust, if fractions are to be learnt from a greater understanding. This study focused on the organization of the children's knowledge on fractions and how it evolved from learning fractions inside the classroom (Empson, 1999).

It is important to pay attention to how students use language to interpret fractions, because it is a way for them to demonstrate their understanding and it can support their mathematical reasoning, ideas, and skills. For example, scholars have shown a study (in this study, the languages that were mainly focused on were Turkish and German) (Prediger & Kuzu, 2019). In the Turkish language, if there is $\frac{3}{5}$ (three fifths), they would say "5 therein 3" or "5 thereof 3", when translated to English (Prediger & Kuzu, 2019). However, in German, it would translate to "3 fifths" or "3 of 5" (Prediger & Kuzu, 2019). As the researchers have stated, "speakers (from different languages) change according to contexts and communication partners" (Prediger & Kuzu, 2019). In this study, bilingual children who were able to speak Turkish and German were able to use the two different perspectives of fractions and made connections between them, which showed "higher learning gains" (Prediger & Kuzu, 2019). The bilingual students discussed the German nuances in Turkish and vice versa, showing that these nuances were incorporated into their overall understanding of fractions (Prediger & Kuzu, 2019). The language is important because students need to know the meanings and how to properly use specialized mathematical terms in order to communicate and negotiate (Muschovich, 2012).

In order to conduct this study, we analyzed the usage of mathematical language from eight third-through-fifth graders (ages seven to twelve) in interview contexts. The aim was to see how children demonstrated their explanations on fractions and what modulation they draw on as they solve problems, while using academic language.

Overview of Methodology

Context of Study

As an undergraduate novice researcher Applied Mathematics student, I, Reshmika Singh, conducted this research study along with the guidance of Dr. Rebecca Ambrose from the School of Education. Due to the COVID-19 pandemic and the need for social distancing, all communication and interviews were given and conducted live video-chat Zoom. Moreover, we collected data by using Zoom and a live digital whiteboard, called Jamboard.

Participants

This study was a qualitative study of children's fraction understanding, therefore the methods involved problem solving interviews with eight children, three boys and five girls followed by an analysis of their responses to the problems. The participants, ages seven to twelve, were recruited by emailing their guardians that were previously interested in participating in a similar research study. During the start of this study, the children were already engaged in online learning at the time.

Data Collection

After recruiting a participant, a meeting needed to be scheduled to interview each student. The meetings were approximately twenty to forty-five minutes long, depending on how many problems that a child completed. We used the Zoom recording option which enabled us to capture video during each interview. The students wrote and showed their work and drawings through a live digital whiteboard website, Jamboard, (for which they were able to share their screen through Zoom.)

We gave each student three to four mathematical problems. The list of problems was given by the mentor, Dr. Rebecca Ambrose, and contained seven fractional equal-sharing problems, and each child was given the same problems (Empson & Levi, 2011, see Appendix A for list of problems). During the interviews, the students were told to show and explain their thoughts and reasonings on how they generated their solutions from each given problem.

The interviewer took an active role in eliciting children's thinking. Occasionally, some students were reticent while being interviewed and did not have much to say. Here are examples of verbatim quotes from a couple of interviews, in order to get the participants to talk:

Quote 1: “Do you want to tell me what you did there?”

Quote 2: “Do you want to tell me what you're thinking?”

Quote 3: “Can you tell me about the things that you have drawn, please?”

Quote 4: “Would you like to explain how you came up with the idea of setting it up?”

Most of the time, the students were communicative when it came to discussing fractional problems. On occasion, a student was not sure how to start a problem, and they were provided with clarification about the given problem or provide suggestions to what might help them get started to solve the problem. For example:

Quote 1: “You want to try labeling the pieces?”

Quote 2: “If you're thinking about one person, how much would that person get?”

Quote 3: “ Okay, you have you, your mom, your brother, and your dad, and if you marked up those tiny cute squares pieces you have, where would those pieces go, and how many would each person get?”

The interviewer also suggested that children draw pictures and write their thoughts down. In Figure A, Student G. drew six squares to see how four people can share them equally.

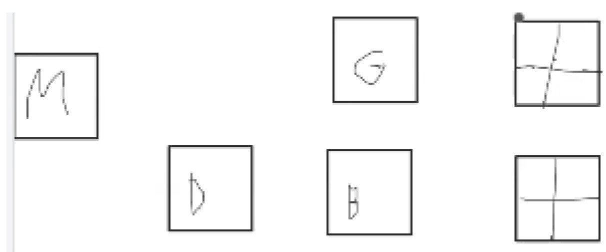

Figure A: Student G showed work for a problem.

After the children completed drawing or showing their work, I asked them questions about it to understand their thinking.

Data Analysis

After each interview, the researcher reviewed the recorded interview and manually typed the transcripts. The transcripts from all participants were used to analyze the academic language they used to explain their work. There were two different ways of looking at the language that children used in their explanations: what fractional terms used and how they discussed their quantities.

To look for particular terms, we identified words that are commonly used when students are learning about or discussing fractions in a classroom-setting. The fractional terms given were: fraction, part, whole, half, split, piece, each, denominator, and every. While going through each transcript, the researcher highlighted and kept count of each child's use of these terms. In addition, we looked at all of the terms used by all 8 participants to see which terms were used most frequently.

Following the usage of terms, we performed a second analysis related to the ways in which students discussed quantities. We analyzed one problem from each child. The problem was: "At Anthony's party, 8 children want to share 6 small cakes equally. How much cake can each child have?" We chose this particular problem, because every participant fully completed it, and this one problem helped us to stay consistent, when collecting data.

We used Thompson's (2010) definition of quantity in our analysis in which a quantity includes an amount along with its units. For example, one of the participants, Student L., stated "*6 cakes,*" during his interview. In this case the amount was stated and the unit was specified, we considered a complete quantity. On occasion, students would not include units when explaining their work. This sometimes made it difficult to know what they were referring to in a problem situation. For example, Student M. stated while talking about her work: "those are four." This verbatim was puzzling because she did not specify what the "four" came from.

Even when children stated quantities, at times it was not clear why these quantities were being discussed. So we coded each statement about quantities according to whether it was "*clear*" or "*vague*" as to why the child was talking about it. By reexamining the previous example, Student L's statement is identified as "*clear*." However, if we could not determine why a quantity was being discussed, then its relevance is indicated as "*vague*."

In order to analyze the quantities from each interview, we took a representative sample of their statements from the transcript that incorporated reasonings and clarifications on how they reached their attempted solutions. The sample contained the students' mathematical thinking done throughout the interview for one problem; researchers decided to focus on one problem to stay consistent with the findings. (No informal utterances were included in the samples.) Figure B shows an example:

Part of Transcript from Student C. :

1) Okay, so each 1 of the small cakes, well lets see… there is 8 of them and 6 small cakes.

2) I know how many…. So if there is 6 of them, then.., and if there is 8 kids… well, lets see.

3) A half, but not just a line and a half…. A line and half, 1 and a half, so 2 fourths like this they

4) would get 2 fourths each. If each kid has 2 fourths, 2 fourths, 2 fourths, 2 fourths, and 2 fourths

(pointing to his drawings).

Magenta = Vague   Blue = Clear

Figure B: A part of Student C's transcript shown to see which indicates *clear* or *vague*.

The following discussion illustrates our approach to analysis. We took a sample where Student C discussed one of the equal-sharing problems. The blue highlight represented "*clear*," which meant that it was why they stated the numbers and quantities. For example, when this student stated "*8 kids,*" we knew that phrase came from the problem given. Therefore, we highlighted it in blue and associated it to be "*clear*." Otherwise, if we did not know where the quantity came from or why it was being discussed, then it was coded as "*vague*." For example, we see that the student stated "*a line and a half*." We could not determine why the students was talking about the line. The importance of the entire methodology used was to substantiate that the students are conscious of their language and can communicate why they were talking about numbers.

Results

After interviewing eight participants, data was collected and we had to split the results into two parts: preliminary and secondary results. The secondary results examined the students' work and drawings while they attempted to reach their solutions.

Preliminary Results

In the preliminary results, researchers kept count of fractional terms used by the students, recognized how precise they were with their quantities, and identified the relevance of the participants' quantities. We calculated the general fractional terms used by each student, during their interviews and totalled them up to see what terms were used often. The significance of these terms was that we were trying to see how frequently the students used those fractional terms, when explaining their process of getting a solution. We came to the conclusion that "each" was used the most, since the problems consisted of finding how much "each" child would get. In addition, "half" was the second most frequently used term by the students. It was possible that some students may have misused "half" during their interview. (Figure A).

Terms	Student T.	Student Z.	Student C.J.	Student J.	Student L.	Student M.	Student F.	Student G.	Totals
Fractions	1	3	0	1	0	1	0	3	**9**
Part(s)	2	11	1	2	9	0	1	0	**17**
Whole	1	5	13	1	2	1	0	0	**23**
Half	15	43	38	6	18	10	8	15	**153**
Split	3	28	2	12	20	2	8	1	**76**
Piece	1	28	0	0	16	0	8	1	**54**
Each	17	72	12	6	33	10	19	8	**177**
Denominator(s)	0	0	0	0	0	0	0	0	**0**

Figure C: Counted the terms used by students, during the interviews.

Figure B shows the participants' level of precision when expressing quantities. We found that about eighty-seven percent of the time, students stated the amounts in their explanations, and they specified the units approximately forty-four percent of the time. For example, if a student stated "6 small cakes," then we saw that an amount (6) is stated and a unit (small cakes) specified. Therefore, while participants specified the amount most of the time, only half the times did they make it clear what unit they were referring to.

Names	Amount Stated	Unit Size Specified
Student L.	6/7....................86%	2/7....................29%
Student Z.	10/12....................83%	6/12....................50%
Student M.	12/15....................80%	7/15....................47%
Student T.	20/23....................87%	6/23....................26%
Student J.	16/18....................89%	13/18....................72%
Student C.	23/26....................88%	9/26....................35%
Student F.	14/17....................82%	8/17....................47%
Student G.	8/8....................100%	4/8....................50%
TOTAL:	**109/126....................87%**	**55/126....................44%**

Figure D: Level of precision in expressing quantities.

In analyzing the relevance of the quantities that students discussed, researchers analyzed why the students discussed quantities, even if they were not precise when stating their quantities. Quantity phrases were considered to be "*clear*" if it was evident why the child was talking about that quantity. Otherwise, quantity phrases were considered to be "*vague*" if it was unnoticeable or unknown why the student was talking about that quantity. For example, when observing Student L., researchers noticed that quantities were "*clear*" five times and "*vague*" twice throughout its explanation for one problem. Therefore, the percentage of the relevance of the quantities that was "*clearly*" discussed was seventy-one and twenty-nine for "*vague*." Overall, quantities discussed in the participants' explanations were clear eighty-five percent of the time. We concluded that the participants tended to make it "*clear*" while they were discussing specific quantities when explaining their strategies. The importance of this chart was to make sure that the students understand and know why they were talking about quantities rather than just arbitrarily operating on numbers.

Names	Clear	Vague	Total	Percentage of Clear	Percentage of Vague
Student L.	5	2	7	71%	29%
Student Z.	12	0	12	100%	0%
Student M.	14	1	15	93%	7%
Student T.	19	4	23	83%	17%
Student J.	17	1	18	94%	6%
Student C.	20	6	26	77%	23%
Student F.	12	5	17	71%	29%
Student G.	8	0	8	100%	0%
TOTAL:	**107**	**19**	**126**	**85%**	**15%**

Figure C: Clear vs Vague Chart.

Secondary Results

In our secondary results, we observed that seven-eighths of the students were not hesitant in explaining their work. We noticed that every student had a certain approach to solving the given problems, but most students used drawings to explain their work.

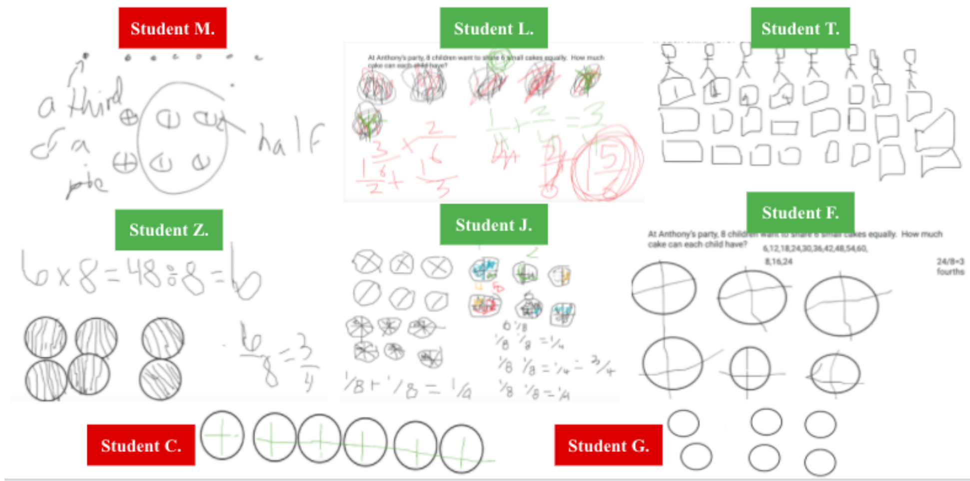

Figure D: Variety of approaches for one particular problem.

Connected to our study, we found that these given equal-sharing fractional problems can be solved in several ways that the participants had done (refer to Figure D). Five of the eight students who were interviewed successfully completed the problem and felt satisfied with their work. However, three of them did not finish the problem. Students M, I, Z, J, F, and Student G started off with drawing six cakes. Then, they split them up into smaller pieces. Student M cut two circles into fourths and four circles into halves and verbally counted how each piece would go to each person. After Student L split up his circles, he managed to compute fractional terms to obtain an answer. Student T drew eight people and forty-eight squares, where each person got three squares. Eventually, Student T counted the pieces for each person, in order to reach her final solution. Student F split each six circles into fourths, and Student Z split them up into eighths. Student J drew circles three times, since he lost count the first two tries. Towards the end, he split the third set of circles into fourths and color coded them to represent each child's equal share of cake. Student F made six circles cut into fourths, which then led her to list out the multiples of six and eight. She found that the least common multiple of six and eight is twenty-four, and she divided it into eighths, resulting in getting a final answer of three fourths. Though Student C's had incomplete work, he drew six circles cut into fourths. Similarly, Student G only managed to draw six circles.

Discussion

After analyzing the data, researchers noticed that the students tended to use the term, "*half*," numerous times. This shows that the students were very comfortable in using that, and the possible reason is that the students were taught how to split a pie in halves, before going into thirds, fourths, and so on. Researchers looked into the participants' level of precision when expressing quantities; and we noticed that less than half of the students specified the units for those amounts, which can happen when being careless while explaining mathematical problems.

However, most of the students were "*clear*" in their explanations, because it was comprehensible and straightforward where and why they stated their quantities. In addition, we observed that the students showed their work and drawing in different ways, during their interviews (refer to Figure D). It could be because they were taught by different teachers or the students' guardians taught them several to solve a fractional equal-sharing problem, which was surprisingly unexpected.

Due to the time constraints during the interviews (twenty to forty minutes in each), some participants struggled to finish to reach their solutions to the three-to-four problems given. A few of them weren't able to finish the third or fourth problem. Another factor of not finishing on time was perhaps that some students were clueless and did not have enough experience and practice with fractions. For example, Student G., an eight years old student in third grade, seemed like she had a difficult time in not being able to finish two problems, especially since she stated that she just started learning fractions at that time. A second participant, Student M. also struggled with the equal-sharing problems, as she stated that her teacher did not teach her fractions at that moment. Furthermore, this research was conducted during the COVID-19 pandemic and when the online classes arose, and it was possible to have unsatisfactory internet connection, especially during one of the interviews with Student C. Moreover, the researchers gave the participants one type of fractional equal-sharing problems, so it could be a reason why some students weren't able to complete it or had struggles with it.

Implications

Attention to language should be central to fraction instruction. Teachers should use correct terminology and help children to develop the terminology they need to explain their thinking. For example, what the "denominators" or "numerators" represent and how they can be used. Teachers need to recognize the importance of students' learning what to call various fractional parts. In our study, during Student M's interview, she drew some circles and cut some of them into halves and fourths, and she stated that each child would get "a third of a pie;" however, she might have meant "three fourths." Thus, children need more opportunities to name fractional quantities. When doing so, students should include the units that they are working with. For example, when that same student stated "a third of a pie," ("pie" is the unit), which indicated the preciseness when discussing quantities. In addition, drawing fractions help students understand fractional quantities. With this, children would be able to recognize and realize what parts of a whole can look like and know how to successfully discuss the fractional concepts. For example, all the participants drew out the fractions to help themselves to thoroughly understand and talk through what each given problems were asking. Furthermore, in our study, it was noticed that there were a variety of strategies to reach a solution in these equal-sharing problems. Learning the different ways of solution strategies would be helpful to students, so they can understand and look at the fractional concepts, in diverse angles. For example, Student F used an approach, where she found the least common multiple, while Student J added fractions to reach a solution. Hence, it is essential to take these measures in order to help students enhance their understanding of fractions.

Conclusion:

At the start of the study, we hypothesized that children would be imprecise when discussing fractions, potentially limiting their understanding. This turned out to be true. While the participants sometimes used academic language in their explanations by using the terms "halves" and "each," they seldom used formal terms such as "whole", "fraction" and "part". Moreover they never used the term "denominator." In addition, they often were imprecise when discussing fractions by excluding the reference to the whole unit. However, most students successfully stated the specific amounts. Additionally, we discovered that some participants were clear in describing how the numbers were related to the problems that were given. In order to build on this study we could expand the number of participants.

The purpose of the research was to have teachers be able to look at our examples and realize better ways to teach fractions to their students, while focusing on their academic language. The most common misconceptions, where children don't often look at the size of the pieces, comes from a heavy emphasis on symbol manipulation rather than a focus on understanding fraction magnitude. According to Fazio and Siegler (2011), "fractions are an important stepping-stone for learning advanced mathematics; they are also commonly used in everyday life." Therefore, language is essential to learn and understand fractions.

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Appendix

A) List of Problems Given to All of the Participants

1. 4 children want to share 6 sandwiches so that each child gets the same amount. How much can each child get?

2. 3 children share 7 small burritos. How much should each child get?

3. At Anthony's party, 8 children want to share 6 small cakes equally. How much cake can each child have?

4. Subway provided 12 sandwiches for a child's birthday party. If there were 18 guests at the party, how much sandwich would each guest get?